NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 2 EXAMINATION 2012-2013

MA3501 **Mathematical Methods in Engineering**

April/May 2013 — Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR** (4) questions and comprises **FIFTEEN** (15) printed pages.

2. Answer **ALL** questions in the examination paper. Marks for each question are indicated at the beginning of the question. The maximum score for this examination is **80** marks.

3. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.

4. Two A4 handwritten double-sided helpsheets are allowed.

5. This ia a **CLOSED BOOK** examination.

Answer **ALL** *questions. Marks for each question are indicated at the beginning of the question.*

Question 1 [20 marks]

(a) The random variable X has the normal distribution $N(1, 20)$. Find the value of a such that $P(X < a) = 2P(X > a)$.

(b) A certain car factory claims that the average hourly salary of its mechanics is \$9.25 with a standard deviation of \$1.55. A random sample of 81 mechanics showed that the average hourly salary of these mechanics was only \$8.95. Test at 1% level of significance whether the average hourly salary of a mechanic is less than \$9.25.

(c) Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} -75 & 25 \\ 50 & -50 \end{bmatrix}.$$

(d) Consider the system of differential equations

$$\begin{bmatrix} \frac{d^2x}{dt^2} \\ \frac{d^2y}{dt^2} \end{bmatrix} = \begin{bmatrix} -75 & 25 \\ 50 & -50 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

Find the scalars u, v, and α such that

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ue^{\alpha t} \\ ve^{\alpha t} \end{bmatrix}$$

is a solution of the system of differential equations.

Question 2 [20 marks]

(a) Use the method of separation of variables to prove that the general solution of the partial differential equation

$$\begin{aligned} u_{tt}(x,t) &= u_{xx}(x,t), & 0<x<1,\ t>0,\\ u(0,t) &= 0, & t>0,\\ u(1,t) &= 0, & t>0, \end{aligned}$$

is

$$u(x,t) = \sum_{n=1}^{\infty}[A_n\cos(n\pi t) + B_n\sin(n\pi t)]\sin(n\pi x),$$

where A_n and B_n are constants for $n = 1, 2, 3, \cdots$.

(b) Solve the boundary value problem

$$\begin{aligned} u_{tt}(x,t) &= u_{xx}(x,t), & 0<x<1,\ t>0,\\ u(0,t) &= 0, & t>0,\\ u(1,t) &= 0, & t>0,\\ u(x,0) &= \sin(\pi x), & 0<x<1,\\ u_t(x,0) &= \sin(2\pi x), & 0<x<1. \end{aligned}$$

Discuss what would happen to the solution $u(x,t)$ if $u_t(x,0)$ is now changed to

$$u_t(x,0) = \begin{cases} \sin(2\pi x), & 0<x<\frac{1}{2}\\ 2, & x=\frac{1}{2}\\ \sin(2\pi x), & \frac{1}{2}<x<1. \end{cases}$$

(c). Consider a long wire fixed at $x = 0$ and stretching to infinity. While at rest, the wire is struck with a hammer so as to impose the initial velocity $\frac{x}{100}$ metre per second over $0 \le x < 1$ and zero elsewhere. The displacement $u(x,t)$ of the wire satisfies the partial differential equation $u_{xx}(x,t) = u_{tt}(x,t)$. Formulate the boundary and initial conditions for the partial differential equation in this model.

Find the function $g(w)$ for all $0 < w < \infty$ such that

$$u(x,t) = \int_0^{\infty} g(w)\sin(wx)\sin(wt)\,dw$$

is the solution of the differential equation.

Question 3 [20 marks]

(a) Solve, by the method of characteristics, the following first order partial differential equation.

$$u_t(x,t) + xu_x(x,t) = 1, \qquad -\infty < x < \infty,\ t > 0,$$
$$u(x,0) = x^2, \qquad -\infty < x < \infty.$$

(b) Let $u(x,t)$ be the solution of the boundary value problem

$$\begin{aligned} u_t(x,t) &= u_{xx}(x,t), && 0 < x < \infty,\ t > 0, \\ u(x,0) &= 0, && 0 < x < \infty, \\ u(0,t) &= \sin t, && t > 0, \\ u(x,t) \text{ is bounded for } & && 0 < x < \infty,\ t > 0. \end{aligned}$$

(i) Find $U(x,s)$ where $U(x,s)$ is the Laplace transform of $u(x,t)$.

(ii) Let $g(x,\tau)$ be the function defined on $0 < x < \infty$, $0 < \tau < \infty$ such that

$$u(x,t) = \int_0^t \sin(t-\tau) g(x,\tau)\, d\tau$$

is the solution of the boundary value problem. Find the function g.

(c) Let $F(w)$ be the Fourier transform of the function $f(x)$ for $-\infty < x, w < \infty$. Given that

$$\int_{-\infty}^{\infty} F(w)\cos(wt)e^{iwx}\, dw = A[f(x+\alpha t) + f(x-\alpha t)]$$

for all $-\infty < x < \infty$, $t > 0$. Find the values of A and α.

Question 4 [20 marks]

(a) Evaluate

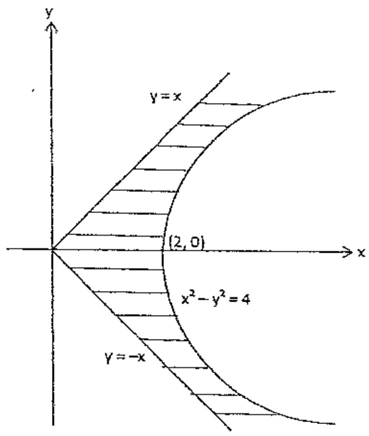
$$\int_\gamma \frac{2}{z(z-3)^2}\, dz,$$

where γ is the anticlockwise oriented circle with centre $(0,0)$ and radius 6.

(b) Let a, b be complex numbers such that $|a| < 1$ and $|b| < 1$. Prove that

$$\left|\frac{a-b}{1-b\bar{a}}\right| < 1.$$

(c) (i) Let D be the region in the right half plane, between the lines $y = x$, $y = -x$ and the curve $x^2 - y^2 = 4$. (D is the shaded region in the figure below.) Find the image of D under the mapping $w(z) = z^2$.



(ii) Solve the Laplace's equation

$$\begin{aligned} \phi_{xx} + \phi_{yy} &= 0 \quad \text{on} \quad D, \\ \phi(x,y) &= 30 \quad \text{on} \quad y = x \text{ and } y = -x, \\ \phi(x,y) &= 20 \quad \text{on} \quad x^2 - y^2 = 4, \end{aligned}$$

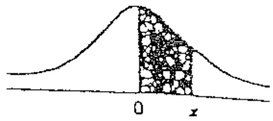
where D is the region defined in (i) above.

END OF PAPER

TURN OVER FOR MATHEMATICAL TABLES

STANDARD NORMAL DISTRIBUTION TABLE

The entries in this table give the areas under the standard normal curve from 0 to z.



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

BOUNDARY VALUE PROBLEMS for $X''(x) + \lambda X(x) = 0$

Boundary conditions	$X(0) = 0$ $X(L) = 0$	$X'(0) = 0$ $X'(L) = 0$	$X(-L) = X(L)$ $X'(-L) = X'(L)$
Eigenvalues λ_n	$(\frac{n\pi}{L})^2$ $n = 1, 2, 3, \ldots$	$(\frac{n\pi}{L})^2$ $n = 0, 1, 2, 3, \ldots$	$(\frac{n\pi}{L})^2$ $n = 0, 1, 2, 3, \ldots$
Eigenfunctions	$\sin \frac{n\pi x}{L}$	$\cos \frac{n\pi x}{L}$	$\sin \frac{n\pi x}{L}$ and $\cos \frac{n\pi x}{L}$
Series	$f(x) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{L}$	$f(x) = \sum_{n=0}^{\infty} A_n \cos \frac{n\pi x}{L}$	$f(x) = \sum_{n=0}^{\infty} A_n \cos \frac{n\pi x}{L}$ $+ \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{L}$
Coefficients	$B_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx$	$A_0 = \frac{1}{L} \int_0^L f(x) dx$ $A_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx$	$A_0 = \frac{1}{2L} \int_{-L}^L f(x) dx$ $A_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx$ $B_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx$

Boundary conditions	$X(0) = 0$ $X'(L) = 0$	$X'(0) = 0$ $X(L) = 0$
Eigenvalues λ_n	$\left[\frac{(2n-1)\pi}{2L}\right]^2$ $n = 1, 2, 3, \ldots$	$\left[\frac{(2n-1)\pi}{2L}\right]^2$ $n = 1, 2, 3, \ldots$
Eigenfunctions	$\sin \frac{(2n-1)\pi x}{2L}$	$\cos \frac{(2n-1)\pi x}{2L}$
Series	$f(x) = \sum_{n=1}^{\infty} B_n \sin \frac{(2n-1)\pi x}{2L}$	$f(x) = \sum_{n=1}^{\infty} B_n \cos \frac{(2n-1)\pi x}{2L}$
Coefficients	$B_n = \frac{2}{L} \int_0^L f(x) \sin \frac{(2n-1)\pi x}{2L} dx$	$B_n = \frac{2}{L} \int_0^L f(x) \cos \frac{(2n-1)\pi x}{2L} dx$

$$\int_0^L \cos\frac{n\pi x}{L} = 0$$

$$\int_{-L}^L \sin\frac{n\pi x}{L} = 0$$

$$\int_0^L \sin\frac{n\pi x}{L}\,\sin\frac{m\pi x}{L}\,dx = \begin{cases} 0, & n \neq m, \\ L/2, & n = m; \end{cases}$$

$$\int_0^L \cos\frac{n\pi x}{L}\,\cos\frac{m\pi x}{L}\,dx = \begin{cases} 0, & n \neq m, \\ L/2, & n = m; \end{cases}$$

$$\int_0^L \sin\frac{(2n-1)\pi x}{2L}\,\sin\frac{(2m-1)\pi x}{2L}\,dx = \begin{cases} 0, & n \neq m, \\ L/2, & n = m; \end{cases}$$

$$\int_0^L \cos\frac{(2n-1)\pi x}{2L}\,\cos\frac{(2m-1)\pi x}{2L}\,dx = \begin{cases} 0, & n \neq m, \\ L/2, & n = m; \end{cases}$$

$$\int x\,\sin(\lambda x)\,dx = \frac{\sin(\lambda x)}{\lambda^2} - \frac{x\cos(\lambda x)}{\lambda}$$

$$\int x\,\cos(\lambda x)\,dx = \frac{\cos(\lambda x)}{\lambda^2} + \frac{x\sin(\lambda x)}{\lambda}$$

$$\int x^2\,\sin(\lambda x)\,dx = \frac{2x\sin(\lambda x)}{\lambda^2} + \frac{(2-\lambda^2 x^2)\cos(\lambda x)}{\lambda^3}$$

$$\int x^2\,\cos(\lambda x)\,dx = \frac{2x\cos(\lambda x)}{\lambda^2} + \frac{(\lambda^2 x^2 - 2)\,\sin(\lambda x)}{\lambda^3}$$

Table of Fourier Transforms

$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \widehat{f}(\omega) e^{ix\omega}\, d\omega$	$\widehat{f}(\omega) = \mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$
1. $\begin{cases} 1 & \text{if } \lvert x\rvert < a \\ 0 & \text{if } \lvert x\rvert > a \end{cases}$	$\sqrt{\frac{2}{\pi}} \frac{\sin a\omega}{\omega}$
2. $\begin{cases} 1 & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases}$	$\frac{i\left(e^{-ib\omega} - e^{ia\omega}\right)}{\sqrt{2\pi}\omega}$
3. $\begin{cases} 1 - \frac{\lvert x\rvert}{a} & \text{if } \lvert x\rvert < a \\ 0 & \text{if } \lvert x\rvert > a \end{cases}$ $a > 0$	$2\sqrt{\frac{2}{\pi}} \frac{\sin^2(\frac{a\omega}{2})}{a\omega^2}$
4. $\begin{cases} x & \text{if } \lvert x\rvert < a \\ 0 & \text{if } \lvert x\rvert > a \end{cases}$ $a > 0$	$i\sqrt{\frac{2}{\pi}} \frac{a\omega\cos(a\omega) - \sin(a\omega)}{\omega^2}$
5. $\begin{cases} \sin x & \text{if } \lvert x\rvert < \pi \\ 0 & \text{if } \lvert x\rvert > \pi \end{cases}$	$i\sqrt{\frac{2}{\pi}} \frac{\sin(\pi\omega)}{\omega^2 - 1}$
6. $\begin{cases} \sin(cx) & \text{if } \lvert x\rvert < b \\ 0 & \text{if } \lvert x\rvert > b \end{cases}$ $a, b > 0$	$i\sqrt{\frac{2}{\pi}} \frac{\omega\cos(b\omega)\sin(ab) - a\cos(ab)\sin(b\omega)}{\omega^2 - a^2}$
7. $\frac{1}{a^2 + x^2}$, $a > 0$	$\sqrt{\frac{\pi}{2}} \frac{e^{-a\lvert\omega\rvert}}{a}$
8. $\frac{x}{a^2 + x^2}$, $a > 0$	$-i\sqrt{\frac{\pi}{2}} \operatorname{sgn}\omega\, e^{-a\lvert\omega\rvert}$
9. $\sqrt{\frac{2}{\pi}} \frac{a}{1 + a^2x^2}$, $a > 0$	$e^{-\frac{\lvert\omega\rvert}{a}}$
10. $\frac{\sin ax}{x}$, $a > 0$	$\begin{cases} \sqrt{\frac{\pi}{2}} & \text{if } \lvert\omega\rvert < a \\ \frac{1}{2}\sqrt{\frac{\pi}{2}} & \text{if } \lvert\omega\rvert = a \\ 0 & \text{if } \lvert\omega\rvert > a \end{cases}$
11. $\frac{4}{\sqrt{2\pi}} \frac{\sin^2(\frac{1}{2}ax)}{ax^2}$, $a > 0$	$\begin{cases} 1 - \frac{\lvert\omega\rvert}{a} & \text{if } \lvert\omega\rvert < a \\ 0 & \text{if } \lvert\omega\rvert > a \end{cases}$
12. $\frac{4}{\sqrt{2\pi}} \frac{\sin^2(ax) - \sin^2(\frac{1}{2}ax)}{ax^2}$, $a > 0$	$\begin{cases} 1 & \text{if } \lvert x\rvert < a \\ (-x + 2a)/a & \text{if } a < x < 2a \\ (x + 2a)/a & \text{if } a < x < 2a \\ 0 & \text{if } \lvert x\rvert > 2a \end{cases}$
13. $e^{-a\lvert x\rvert}$, $a > 0$	$\sqrt{\frac{2}{\pi}} \frac{a}{a^2 + \omega^2}$
14. $\begin{cases} e^{-ax} & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$, $a > 0$	$\frac{1}{\sqrt{2\pi}} \frac{1}{a + i\omega}$
15. $\begin{cases} 0 & \text{if } x > 0 \\ e^{ax} & \text{if } x < 0 \end{cases}$, $a > 0$	$\frac{1}{\sqrt{2\pi}} \frac{1}{a - i\omega}$
16. $\lvert x\rvert^n e^{-a\lvert x\rvert}$, $a > 0$, $n > 0$	$\frac{\Gamma(n+1)}{\sqrt{2\pi}} \left(\frac{1}{(a - i\omega)^{1+n}} + \frac{1}{(a + i\omega)^{1+n}} \right)$

Table of Fourier Transforms (continued)

$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \widehat{f}(\omega) e^{ix\omega}\, d\omega$	$\widehat{f}(\omega) = \mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$
17. $e^{-\frac{a}{2}x^2}$, $a > 0$	$\frac{1}{\sqrt{a}} e^{-\frac{\omega^2}{2a}}$
18. e^{-ax^2}, $a > 0$	$\frac{1}{\sqrt{2a}} e^{-\frac{\omega^2}{4a}}$
19. $xe^{-\frac{a}{2}x^2}$, $a > 0$	$\frac{-i\omega}{a^{3/2}} e^{-\frac{\omega^2}{2a}}$
20. $x^2 e^{-\frac{a}{2}x^2}$, $a > 0$	$\frac{a-\omega^2}{a^{5/2}} e^{-\frac{\omega^2}{2a}}$
21. $x^3 e^{-\frac{a}{2}x^2}$, $a > 0$	$\frac{-i\omega(3a-\omega^2)}{a^{7/2}} e^{-\frac{\omega^2}{2a}}$
22. $e^{-\frac{x^2}{2}} H_n(x)$, H_n, nth Hermite polynomial	$(-1)^n i^n e^{\frac{-\omega^2}{2}} H_n(\omega)$
23. $J_0(x)$, Bessel function of order 0	$\begin{cases} \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{1-\omega^2}} & \text{if } \lvert\omega\rvert < 1 \\ 0 & \text{if } \lvert\omega\rvert > 1 \end{cases}$
24. $J_n(x)$, Bessel function of order $n \geq 0$	$\begin{cases} \sqrt{\frac{2}{\pi}} \frac{(-i)^n}{\sqrt{1-\omega^2}} T_n(\omega) & \text{if } \lvert\omega\rvert < 1 \\ 0 & \text{if } \lvert\omega\rvert > 1 \end{cases}$ T_n, Chebyshev polynomial of degree n.

Special Transforms

25. $\mathcal{F}(\delta_0(x))(\omega) = \frac{1}{\sqrt{2\pi}}$	27. $\mathcal{F}(\sqrt{\frac{2}{\pi}} \frac{1}{x})(\omega) = -i \operatorname{sgn} \omega$
26 $\mathcal{F}(\delta_0(x-a))(\omega) = \frac{1}{\sqrt{2\pi}} e^{-ia\omega}$	28. $\mathcal{F}(e^{iax})(\omega) = \sqrt{2\pi}\, \delta_0(\omega - a)$

Operational Properties

29. $\mathcal{F}(af + bg)(\omega) = a\mathcal{F}(f) + b\mathcal{F}(g)$	36. $\mathcal{F}(f\,g)(\omega) = \mathcal{F}(f) * \mathcal{F}(g)(\omega)$
30. $\mathcal{F}(f')(\omega) = i\omega \mathcal{F}(f)(\omega)$	37. $\mathcal{F}(f(x-a))(\omega) = e^{-ia\omega} \mathcal{F}(f)(\omega)$
31. $\mathcal{F}(f'')(\omega) = -\omega^2 \mathcal{F}(f)(\omega)$	38. $\mathcal{F}(e^{iax} f(x))(\omega) = \mathcal{F}(f)(\omega - a)$
32. $\mathcal{F}(f^{(n)})(\omega) = (i\omega)^n \mathcal{F}(f)(\omega)$	39. $\mathcal{F}(\cos(ax) f(x))(\omega) = \frac{\mathcal{F}(f)(\omega-a) + \mathcal{F}(f)(\omega+a)}{2}$
33. $\mathcal{F}(xf(x))(\omega) = i \frac{d}{d\omega} \mathcal{F}(f)(\omega)$	40. $\mathcal{F}(\sin(ax) f(x))(\omega) = \frac{\mathcal{F}(f)(\omega-a) - \mathcal{F}(f)(\omega+a)}{2i}$
34. $\mathcal{F}(x^n f(x))(\omega) = i^n \frac{d^n}{d\omega^n} \mathcal{F}(f)(\omega)$	41. $\mathcal{F}(f(ax))(\omega) = \frac{1}{\lvert a\rvert} \mathcal{F}(f)(\frac{\omega}{a})$, $a \neq 0$
35. $\mathcal{F}(f * g)(\omega) = \mathcal{F}(f)(\omega) \mathcal{F}(g)(\omega)$	42. $f(x) = \mathcal{F}(\widehat{f})(-x)$, $\mathcal{F}(\mathcal{F}(f)) = f(-x)$

Table of Fourier Cosine Transforms

$f(x)=\sqrt{\frac{2}{\pi}}\int_0^\infty \mathcal{F}_c(f)(\omega)\cos\omega x\,d\omega,\quad 0<x<\infty$	$\mathcal{F}_c(f)(\omega)=\widehat{f}_c(\omega)=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos\omega x\,dx,\quad 0\le\omega<\infty$
1. $\begin{cases}1 & \text{if } 0<x<a\\ 0 & \text{otherwise}\end{cases}$	$\sqrt{\frac{2}{\pi}}\frac{\sin a\omega}{\omega}$
2. $e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{a}{a^2+\omega^2}$
3. $x\,e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{a^2-\omega^2}{(a^2+\omega^2)^2}$
4. $e^{-a x^2/2},\quad a>0$	$\frac{1}{\sqrt{a}}e^{-\omega^2/2a}$
5. $\cos ax\,e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{a\omega^2+2a^3}{4a^4+\omega^4}$
6. $\sin ax\,e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{2a^3-a\omega^2}{4a^4+\omega^4}$
7. $\frac{a}{a^2+x^2},\quad a>0$	$\sqrt{\frac{\pi}{2}}e^{-a\omega}$
8. $x^p,\quad 0<p<1$	$\sqrt{\frac{2}{\pi}}\frac{\Gamma(p)\cos(p\omega/2)}{\omega^p}$
9. $\begin{cases}\cos x & \text{if } 0<x<a\\ 0 & \text{otherwise}\end{cases}$	$\frac{1}{\sqrt{2\pi}}\left[\frac{\sin a(1-\omega)}{1-\omega}+\frac{\sin a(1+\omega)}{1+\omega}\right]$
Operational Properties	
10. $\alpha f(x)+\beta g(x)$	$\alpha\mathcal{F}_c(f)(\omega)+\beta\mathcal{F}_c(g)(\omega)$
11. $f(ax),\quad a>0$	$\frac{1}{a}\widehat{f}_c(\frac{\omega}{a})$
12. $f'(x)$	$\omega\widehat{f}_s(\omega)-\sqrt{\frac{2}{\pi}}f(0)$
13. $f''(x)$	$-\omega^2\widehat{f}_c(\omega)-\sqrt{\frac{2}{\pi}}f'(0)$
14. $xf(x)$	$\left[\widehat{f}_s\right]'(\omega)$
15. $\mathcal{F}_c(\mathcal{F}_c f)$	f

Table of Fourier Sine Transforms

	$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty \mathcal{F}_s(f)(\omega) \sin \omega x \, d\omega$, $0 < x < \infty$	$\mathcal{F}_s(f)(\omega) = \widehat{f}_s(\omega) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin \omega x \, dx$, $0 \leqslant \omega < \infty$
1.	$\begin{cases} 1 & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$	$\sqrt{\frac{2}{\pi}} \frac{1-\cos a\omega}{\omega}$
2.	$e^{-ax}, \quad a > 0$	$\sqrt{\frac{2}{\pi}} \frac{\omega}{a^2+\omega^2}$
3.	$x e^{-ax}, \quad a > 0$	$\sqrt{\frac{2}{\pi}} \frac{2a\omega}{(a^2+\omega^2)^2}$
4.	$\frac{e^{-ax}}{x}, \quad a > 0$	$\sqrt{\frac{2}{\pi}} \tan^{-1} \frac{\omega}{a}$
5.	$\frac{1}{2} x e^{-a x^2}, \quad a > 0$	$\frac{\omega}{a^{3/2}} e^{-\omega^2/2a}$
6.	$\cos ax \, e^{-ax}, \quad a > 0$	$\sqrt{\frac{2}{\pi}} \frac{\omega^3}{4a^4+\omega^4}$
7.	$\sin ax \, e^{-ax}, \quad a > 0$	$\sqrt{\frac{2}{\pi}} \frac{2a^2\omega}{4a^4+\omega^4}$
8.	$\frac{x}{a^2+x^2}, \quad a > 0$	$\sqrt{\frac{\pi}{2}} e^{-a\omega}$
9.	$x^{p-1}, \quad 0 < p < 1$	$\sqrt{\frac{2}{\pi}} \frac{\Gamma(p) \cos(\pi p/2)}{\omega^p}$
10.	$\begin{cases} \sin x & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$	$\frac{1}{\sqrt{2\pi}} \left[\frac{\sin a(1-\omega)}{1-\omega} - \frac{\sin a(1+\omega)}{1+\omega} \right]$
	Operational Properties	
11.	$\alpha f(x) + \beta g(x)$	$\alpha \mathcal{F}_s(f)(\omega) + \beta \mathcal{F}_s(g)(\omega)$
12.	$f(ax), \quad a > 0$	$\frac{1}{a} \widehat{f}_s(\frac{\omega}{a})$
13.	$f'(x)$	$-\omega \widehat{f}_c(\omega)$
14.	$f''(x)$	$-\omega^2 \widehat{f}_s(\omega) + \sqrt{\frac{2}{\pi}} \omega f(0)$
15.	$x f(x)$	$-\left[\widehat{f}_c\right]'(\omega)$
16.	$\mathcal{F}_s(\mathcal{F}_s f)$	f

Table of Laplace Transforms

	$f(t), \quad t \geq 0$	$F(s) = \mathcal{L}(f)(s) = \int_0^\infty f(t)e^{-st}\,dt,$
1.	1	$\frac{1}{s}, \quad s > 0$
2.	$t^n, \quad n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}}, \quad s > 0$
3.	$t^a \ (a > -1)$	$\frac{\Gamma(a+1)}{s^{a+1}}, \quad s > 0$
4.	e^{at}	$\frac{1}{s-a}, \quad s > a$
5.	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
6.	$\frac{e^{at}-e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}, \quad s > \max(a, b)$
7.	$\frac{ae^{at}-be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}, \quad s > \max(a, b)$
8.	$\sin kt$	$\frac{k}{s^2+k^2}, \quad s > 0$
9.	$\cos kt$	$\frac{s}{s^2+k^2}, \quad s > 0$
10.	$e^{at} \sin kt$	$\frac{k}{(s-a)^2+k^2}, \quad s > a$
11.	$e^{at} \cos kt$	$\frac{s-a}{(s-a)^2+k^2}, \quad s > a$
12.	$t \sin kt$	$\frac{2ks}{(s^2+k^2)^2}, \quad s > 0$
13.	$t \cos kt$	$\frac{s^2-k^2}{(s^2+k^2)^2}, \quad s > 0$
14.	$\frac{1}{2a^3}(\sin at - at \cos at)$	$\frac{1}{(s^2+a^2)^2}, \quad s > 0$
15.	$\sinh kt$	$\frac{k}{s^2-k^2}, \quad s > \|k\|$
16.	$\cosh kt$	$\frac{s}{s^2-k^2}, \quad s > \|k\|$
17.	$e^{at} \sinh kt$	$\frac{k}{(s-a)^2-k^2} \quad s > a + \|k\|$
18.	$e^{at} \cosh kt$	$\frac{s-a}{(s-a)^2-k^2} \quad s > a + \|k\|$
19.	$t \sinh kt$	$\frac{2ks}{(s^2-k^2)^2}, \quad s > \|k\|$
20.	$t \cosh kt$	$\frac{s^2+k^2}{(s^2-k^2)^2}, \quad s > \|k\|$
21.	$\frac{1}{2k^3}(kt \cosh kt - \sinh kt)$	$\frac{1}{(s^2-k^2)^2}, \quad s > \|k\|$

Table of Laplace Transforms (continued)

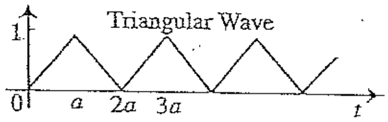
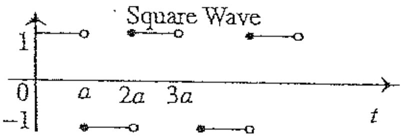
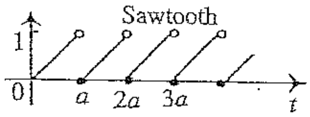
	$f(t), \quad t \ge 0$	$F(s) = \mathcal{L}(f)(s) = \int_0^\infty f(t)e^{-st}\,dt,$
22.	$\delta_0(t-t_0), \quad t_0 \ge 0$	$e^{-t_0 s}, \quad s > 0$
23.	$\mathcal{U}_0(t-a) = \begin{cases} 0 & \text{if } t < a \\ 1 & \text{if } t \ge a \end{cases} \quad (a > 0)$	$\frac{e^{-as}}{s}, \quad s > 0$
24.	$f(t+T) = f(t) \quad (T > 0)$	$\frac{1}{1-e^{-Ts}} \int_0^T e^{-st} f(t)\,dt$
25.	$f(t+T) = -f(t) \quad (T > 0)$	$\frac{1}{1+e^{-Ts}} \int_0^T e^{-st} f(t)\,dt$
26.		$\frac{1}{as^2}\left[\frac{1-e^{-as}}{1+e^{-as}}\right] = \frac{1}{as^2}\tanh\left(\frac{as}{2}\right), \quad s > 0$
27.		$\frac{1}{s}\left[\frac{1-e^{-as}}{1+e^{-as}}\right] = \frac{1}{s}\tanh\left(\frac{as}{2}\right), \quad s > 0$
28.		$\frac{1}{as^2} - \frac{e^{-as}}{s(1-e^{-as})}, \quad s > 0$
29.	$\frac{\sin at}{t}$	$\tan^{-1}\left(\frac{a}{s}\right), \quad s > 0$
30.	$J_0(at)$	$\frac{1}{\sqrt{s^2+a^2}}, \quad s > 0$
31.	$J_0(a\sqrt{t})$	$\frac{e^{-a^2/4s}}{s}, \quad s > 0$
32.	$t^p J_p(at) \quad (p > -\frac{1}{2})$	$\frac{2^p a^p \Gamma(p+\frac{1}{2})}{\sqrt{\pi}(s^2+a^2)^{p+\frac{1}{2}}}, \quad s > 0$
33.	$\frac{\sqrt{\pi}}{\Gamma(k)}\left(\frac{t}{2a}\right)^{k-\frac{1}{2}} J_{k-\frac{1}{2}}(at) \quad (k > 0)$	$\frac{1}{(s^2+a^2)^k}, \quad s > 0$
34.	$\frac{\sqrt{\pi}}{\Gamma(k)} a \left(\frac{t}{2a}\right)^{k-\frac{1}{2}} J_{k-\frac{3}{2}}(at) \quad (k > \frac{1}{2})$	$\frac{s}{(s^2+a^2)^k}, \quad s > 0$
35.	$2\sum_{m=1}^{n} \binom{2n-m-1}{n-1} \frac{t^{m-1}\cos(at - \frac{m\pi}{2})}{(2a)^{2n-m}(m-1)!}$ (n an integer ≥ 1)	$\frac{1}{(s^2+a^2)^n}, \quad s > 0$
36.	$\frac{1}{(n-1)!}\sum_{m=1}^{n-1} \frac{(2n-m-3)!}{(m-1)!(n-m-1)!} \frac{t^m \cos(at - \frac{m\pi}{2})}{(2a)^{2n-m-2}}$ (n an integer ≥ 2)	$\frac{s}{(s^2+a^2)^n}, \quad s > 0$

Table of Laplace Transforms (continued)

	$f(t), \quad t \geq 0$	$F(s) = \mathcal{L}(f)(s) = \int_0^\infty f(t)e^{-st}\,dt,$
37.	$\operatorname{erf}(at) \quad (a > 0)$	$\frac{1}{s}\, e^{s^2/4a^2} \operatorname{erfc}\left(\frac{s}{2a}\right), \quad s > 0$
38.	$\operatorname{erf}(a\sqrt{t})$	$\frac{a}{s\sqrt{s+a^2}}, \quad s > 0$
39.	$e^{-a^2t^2} \quad (a > 0)$	$\frac{\sqrt{\pi}}{2a} e^{s^2/4a^2} \operatorname{erfc}(\frac{s}{2a}), \quad s > 0$
40.	$\frac{1}{\sqrt{\pi t}} e^{-a^2/4t} \quad (a \geq 0)$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}, \quad s > 0$
41.	$\frac{a}{2\sqrt{\pi} t^{3/2}} e^{-a^2/4t} \quad (a > 0)$	$e^{-a\sqrt{s}}, \quad s > 0$
42.	$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right) \quad (a \geq 0)$	$\frac{1}{s}\, e^{-a\sqrt{s}}, \quad s > 0$
	Operational Properties	
43.	$\alpha f(t) + \beta g(t)$	$\alpha F(s) + \beta G(s)$
44.	$f'(t)$	$sF(s) - f(0)$
45.	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
46.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0)$
47.	$-tf(t)$	$F'(s)$
48.	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
49.	$\int_0^t f(\tau)\,d\tau$	$\frac{1}{s} F(s), \quad s > 0$
50.	$\int_0^t \int_0^\tau f(\rho)\,d\rho d\tau$	$\frac{1}{s^2} F(s), \quad s > 0$
51.	$\frac{f(t)}{t}$	$\int_s^\infty F(u)\,du$
52.	$\frac{f(t)}{t^2}$	$\int_s^\infty \int_\sigma^\infty F(u)\,du d\sigma$
53.	$\mathcal{U}(t-a)f(t-a) \quad (a > 0)$	$e^{-as}F(s)$
54.	$e^{at}f(t)$	$F(s-a)$
55.	$f(ct) \quad (c > 0)$	$\frac{1}{c}F(\frac{s}{c})$
56.	$f * g(t) = \int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$