Matriculation Number:

NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF SCIENCE

SEMESTER 2 EXAMINATION 2008-2009

MA1506 MATHEMATICS II

April 2009 Time allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. **Write down your matriculation number neatly in the space provided above.** This booklet (and only this booklet) will be collected at the end of the examination. Do not insert any loose pages in the booklet.
2. This examination paper consists of **EIGHT (8)** questions and comprises **THIRTY THREE (33)** printed pages.
3. Answer **ALL** questions. For each question, write your answer in the box and your working in the space provided inside the booklet following that question.
4. The marks for each question are indicated at the beginning of the question.
5. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.

For official use only. Do not write below this line.

Question	1	2	3	4	5	6	7	8
Marks								

Question 1 (a) [5 marks]

Let $y(x)$ be the solution of the initial value problem

$$2x - y\frac{dy}{dx} = 0, \quad y \geq 0, \quad \text{and} \quad y(0) = 2.$$

Find the value of $y(4)$.

Answer 1(a)	6

(Show your working below and on the next page.)

$$y\,dy = 2x\,dx$$

$$\frac{1}{2}y^2 = x^2 + C$$

$$y(0) = 2 \Rightarrow C = 2$$

$$\therefore \quad y^2 = 2x^2 + 4$$

$$y(4) = \sqrt{2(4)^2 + 4}$$

$$= \underline{\underline{6}}$$

Question 1 (b) [5 marks]

A hall of volume 1000 cubic metres contains air with 0.001% of carbon monoxide. At time $t = 0$ the ventilation system starts blowing in air which contains 2% of carbon monoxide by volume. If the ventilation system blows in and extracts air at a rate of 0.3 cubic metres per minute, how long will it take for the air in the hall to contain 0.025% of carbon monoxide? Give your answer in minutes correct to two decimal places.

Answer 1(b)	40.26 minutes

(Show your working below and on the next page.)

Let $x\ m^3$ = volume of CO at time t.

$$\frac{dx}{dt} = (0.3)(0.02) - (0.3)\frac{x}{1000}$$

$$\therefore \frac{dx}{dt} = 0.0003(20-x)$$

$$\frac{dx}{20-x} = 0.0003\,dt \Rightarrow -\ln|20-x| = 0.0003t + C$$

$$\therefore 20-x = Ae^{-0.0003t}$$

$$t=0,\ x = 1000(0.001\%) = 0.01 \Rightarrow A = 19.99$$

$$\therefore x = 20 - 19.99e^{-0.0003t}$$

$$x = 1000(0.025\%) = 0.25 \Rightarrow 19.99e^{-0.0003t} = 19.75$$

$$\therefore t = \frac{\ln(19.75/19.99)}{-0.0003} \approx \underline{\underline{40.26 \text{ minutes}}}$$

Question 2 (a) [5 marks]

At 11am, a cup of coffee at 86^oC is placed in an aircon room which has a temperature of 23^oC. The aircon is immediately switched off and the room warms up at an uniform rate to 32^oC at 12 noon. Assume that at any time between 11am and 12 noon, the rate of change of temperature T of the coffee satisfies the equation:

$$\frac{dT}{dt} = -(T - T_{env})$$

where T_{env} is the room temperature at that time and the unit of time is measured in hours, find the temperature of the coffee at 11:30am. Give your answer in ^oC correct to two decimal places.

Answer 2(a)	62.17

(Show your working below and on the next page.)

$\because$ Tem goes from 23°C to 32°C uniformly in 1 hour

$\therefore$ Tem $= 23 + 9t$

$$\therefore \frac{dT}{dt} = -(T - 23 - 9t) = -T + 23 + 9t$$

$$\therefore \frac{dT}{dt} + T = 23 + 9t$$

Integrating factor $R = e^{\int dt} = e^{t}$

$$\therefore T = e^{-t}\int e^{t}(23 + 9t)\,dt = e^{-t}\{23e^{t} + \int 9te^{t}\,dt\}$$

$$= e^{-t}\{23e^{t} + 9te^{t} - 9e^{t} + c\}$$

$T(0) = 86 \Rightarrow c = 72 \Rightarrow T = 14 + 9t + 72e^{-t}$

$$\therefore T(\tfrac{1}{2}) = 14 + \frac{9}{2} + 72e^{-0.5} \approx \underline{\underline{62.17}}\ ^{\circ}C$$

Question 2 (b) [5 marks]

Solve the differential equation

$$y'' - y = 2e^x$$

with the initial conditions

$$y(0) = 2, \quad y'(0) = 1.$$

Answer 2(b)	$y = e^x + e^{-x} + xe^x$

(Show your working below and on the next page.)

$\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$

Try $y = (Ax+B)e^x \Rightarrow y' = Ae^x + (Ax+B)e^x$

$\Rightarrow y'' = 2Ae^x + (Ax+B)e^x$

$\therefore 2Ae^x = 2e^x \Rightarrow A = 1$

$\therefore y = c_1 e^x + c_2 e^{-x} + xe^x$

$y' = c_1 e^x - c_2 e^{-x} + e^x + xe^x$

$y(0) = 2, \ y'(0) = 1 \Rightarrow c_1 = 1, \ c_2 = 1.$

$\therefore \underline{\underline{y = e^x + e^{-x} + xe^x}}$

Question 3 (a) [5 marks]

Consider the equation

$$\ddot{x} = \sec^2 x - 2, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

Find the equation of the phase curve which satisfies the initial conditions $x(0) = 0$, $y(0) = 1$, where $y = \dot{x}$.

Answer 3(a)	$y^2 = 2\tan x - 4x + 1$

(Show your working below and on the next page.)

$$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}\dot{x}^2\right)$$

$$\frac{d}{dx}\left(\frac{1}{2}y^2\right) = \sec^2 x - 2$$

$$\frac{1}{2}y^2 = \tan x - 2x + C$$

$$x=0,\ y=1 \Rightarrow C = \frac{1}{2}$$

$$\therefore\ y^2 = 2\tan x - 4x + 1$$

Question 3 (b) [5 marks]

You have 20000 bugs in a bottle. It is known that this bug population is given by a logistic model with a birth rate per capita of 25% per day. After a long time, you find that the population has attained an equilibrium of 100000 bugs. Then you start an experiment with an antibiotic which kills 5000 bugs a day. Find approximately how many bugs you will still have after another very long period of time. Give your answer correct to the nearest ten.

Answer 3(b)	72360

(Show your working below and on the next page.)

$B = 25\% = 0.25, \quad \frac{B}{S} = 100000 \Rightarrow S = \frac{0.25}{100000}$

$$\frac{dN}{dt} = 0.25N - \frac{0.25}{100000}N^2 - 5000$$

$$0.25N - \frac{0.25}{100000}N^2 - 5000 = 0$$

$$\Rightarrow N^2 - 100000N + 2000000000 = 0$$

$$N = \frac{10^5 \pm \sqrt{10^{10} - 8\times 10^9}}{2} = \frac{10^5 \pm \sqrt{20}\times 10^4}{2}$$

$$\approx 27639.5 \text{ or } 72360.5$$

$$\therefore N \approx \underline{\underline{72360}}$$

Question 4 (a) [5 marks]

Find and classify (i.e. determine whether each one is stable or unstable) all the equilibrium solutions of the differential equation

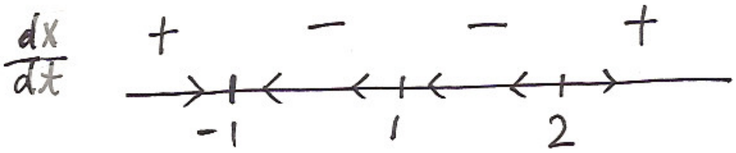
$$\frac{dx}{dt} = x^4 - 3x^3 + x^2 + 3x - 2.$$

(Hint: $(x-2)$ is a factor of the right hand side.)

Answer 4(a)	$x=-1$ stable $x=1$ unstable $x=2$ unstable

(Show your working below and on the next page.)

$$\frac{dx}{dt} = (x-1)^2(x+1)(x-2)$$



$$\begin{cases} x=-1 & \text{stable} \\ x=1 & \text{unstable} \\ x=2 & \text{unstable} \end{cases}$$

Question 4 (b) [5 marks]

Compute the inverse Laplace transform of

$$F(s) = \frac{s-2}{s^2 - 4s + 13}.$$

Answer 4(b)	$e^{2t} \cos 3t$

(Show your working below and on the next page.)

$$\mathcal{L}^{-1}\left(\frac{s-2}{s^2-4s+13}\right) = \mathcal{L}^{-1}\left(\frac{s-2}{(s-2)^2+3^2}\right)$$

$$= \underline{\underline{e^{2t}\cos 3t}}$$

Question 5 (a) [5 marks]

In an RLC circuit of inductance 1 henry, resistance 3 ohms, capacitance $\frac{1}{2}$ farads and voltage V volts, it is known that the charge of the capacitor Q satisfies the equation

$$\frac{d^2Q}{dt^2} + 3\frac{dQ}{dt} + 2Q = V.$$

At time $t = 0$ seconds, you observe that there is no voltage applied to the circuit and that both Q and $\frac{dQ}{dt}$ are zero at that time. At time $t = 2$ seconds, you apply a constant voltage of 2 volts to the circuit and then you switch off the voltage at time $t = 4$ seconds. Find the value of Q at time $t = 5$ seconds. Give your answer correct to three decimal places.

Answer 5(a)	0.503

(Show your working below and on the next page.)

$$\begin{cases} \dfrac{d^2Q}{dt^2} + 3\dfrac{dQ}{dt} + 2Q = 2\{u(t-2) - u(t-4)\} \\ Q(0) = \dfrac{dQ}{dt}(0) = 0 \end{cases}$$

$$L(Q)\{s^2+3s+2\} = \frac{2}{s}\{e^{-2s} - e^{-4s}\}$$

$$L(Q) = \frac{2}{s(s+1)(s+2)}\{e^{-2s} - e^{-4s}\}$$

Let $\dfrac{2}{s(s+1)(s+2)} = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{C}{s+2}$

(More working space for Question 5(a))

$$\therefore 2 = A(s+1)(s+2) + Bs(s+2) + Cs(s+1)$$

$$s=0 \Rightarrow A = 1$$

$$s=-1 \Rightarrow B = -2$$

$$s=-2 \Rightarrow C = 1$$

$$\therefore \mathcal{L}(Q) = \left\{\frac{1}{s} + \frac{-2}{s+1} + \frac{1}{s+2}\right\}\left\{e^{-2s} - e^{-4s}\right\}$$

$$= \left\{\frac{1}{s} + \frac{-2}{s+1} + \frac{1}{s+2}\right\} e^{-2s}$$

$$- \left\{\frac{1}{s} + \frac{-2}{s+1} + \frac{1}{s+2}\right\} e^{-4s}$$

$$Q = \left\{1 - 2e^{-t+2} + e^{-2t+4}\right\} u(t-2)$$

$$- \left\{1 - 2e^{-t+4} + e^{-2t+8}\right\} u(t-4)$$

$$\therefore Q(5) = -2e^{-3} + e^{-6} + 2e^{-1} - e^{-2}$$

$$\approx \underline{\underline{0.503}}$$

Question 5 (b) [5 marks]

Morphine in the blood decomposes with a half life of 2.9 hours. Suppose a doctor injects 60 mg of morphine into a patient and does it again 6 hours later. Find the amount of morphine in mg in the patient's blood at the seventh hour. Give your answer correct to one decimal place. (You may assume that there is no morphine in the patient's blood prior to the first injection.)

Answer 5(b)	58.5

(Show your working below and on the next page.)

$$\frac{dM}{dt} = -kM \text{ and half-life} = 2\cdot 9 \text{ hours}$$

$$\Rightarrow k = \frac{\ln 2}{2\cdot 9}$$

$$\begin{cases} \dfrac{dM}{dt} = -kM + 60\,\delta(t) + 60\,\delta(t-6) \\ M(0) = 0 \end{cases}$$

$$sL(M) = -k\,L(M) + 60 + 60e^{-6s}$$

$$L(M) = \frac{60}{s+k} + \frac{60}{s+k}e^{-6s}$$

$$M = 60e^{-kt} + 60e^{-k(t-6)}u(t-6)$$

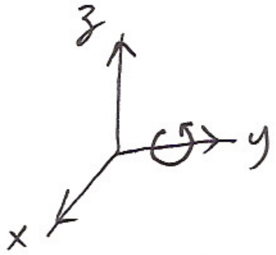
$$M(7) = 60e^{-\frac{7\ln 2}{2\cdot 9}} + 60e^{-\frac{\ln 2}{2\cdot 9}} \approx \underline{\underline{58.5}}$$

Question 6 (a) [5 marks]

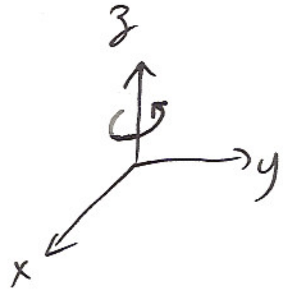
Where will the point $(1, 2, 3)$ move to if we rotate 90^o around the y-axis according to the right-hand rule and then rotate 90^o around the z-axis according to the right-hand rule?

Answer 6(a)	$(-2, 3, -1)$

(Show your working below and on the next page.)



$$\vec{i} \to -\vec{k} \qquad \vec{j} \to \vec{j} \qquad \vec{k} \to \vec{i} \quad \Rightarrow \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$



$$\vec{i} \to \vec{j} \qquad \vec{j} \to -\vec{i} \qquad \vec{k} \to \vec{k} \quad \Rightarrow \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}}}$$

Question 6 (b) [5 marks]

A country has two main sectors in its economy: Agriculture (A) and Manufacturing (M). The demand for A and M are \$120 millions per year and \$100 millions per year respectively. It costs 40 cents of A to generate one dollar of A. It also costs 30 cents of M to generate one dollar of A. It costs 30 cents of A to generate one dollar of M. It also costs 60 cents of M to generate one dollar of M. Formulate this as a Leontief input-output model and find the production in millions of dollars per year for A and M.

Answer 6(b)	$A = 520$ $B = 640$

(Show your working below and on the next page.)

$$\begin{cases} A = 0.4A + 0.3M + 120 \\ M = 0.3A + 0.6M + 100 \end{cases}$$

$$\begin{pmatrix} A \\ M \end{pmatrix} = \begin{pmatrix} 0.4 & 0.3 \\ 0.3 & 0.6 \end{pmatrix} \begin{pmatrix} A \\ M \end{pmatrix} + \begin{pmatrix} 120 \\ 100 \end{pmatrix}$$

$$\begin{pmatrix} 0.6 & -0.3 \\ -0.3 & 0.4 \end{pmatrix} \begin{pmatrix} A \\ M \end{pmatrix} = \begin{pmatrix} 120 \\ 100 \end{pmatrix}$$

$$\begin{pmatrix} A \\ M \end{pmatrix} = \begin{pmatrix} 0.6 & -0.3 \\ -0.3 & 0.4 \end{pmatrix}^{-1} \begin{pmatrix} 120 \\ 100 \end{pmatrix}$$

$$= \frac{1}{0.15} \begin{pmatrix} 0.4 & 0.3 \\ 0.3 & 0.6 \end{pmatrix} \begin{pmatrix} 120 \\ 100 \end{pmatrix} = \begin{pmatrix} 520 \\ 640 \end{pmatrix}$$

Question 7 (a) [5 marks]

Let a and b denote two constants. It is known that the matrix

$$\begin{pmatrix} 5 & 2 & a \\ \frac{32}{5} & 3 & \frac{-12}{5} \\ 10 & b & -2 \end{pmatrix}$$

has three distinct eigenvalues, that one of the eigenvalues is equal to 2, and that its determinant is equal to −90. Find the other two eigenvalues.

Answer 7(a)	$-5, 9$

(Show your working below and on the next page.)

$$2+\lambda_1+\lambda_2 = Tr = 6 \Rightarrow \lambda_1+\lambda_2 = 4$$

$$2\lambda_1\lambda_2 = det = -90 \Rightarrow \lambda_1\lambda_2 = -45$$

$\therefore \lambda_1, \lambda_2$ are roots of $\lambda^2 - 4\lambda - 45$

$$= (\lambda-9)(\lambda+5)$$

$$\therefore \lambda_1, \lambda_2 = \underline{\underline{-5, 9}}$$

Question 7 (b) [5 marks]

A certain colony of rabbits is being attacked by foxes. Both rabbits and foxes are dying out due to a severe drought. Let R_n and F_n denote the number of rabbits and the number of foxes respectively in year n. It is known that these numbers satisfy the equations

$$\begin{cases} R_{k+1} = \frac{1}{2}R_k - \frac{1}{4}F_k \\ F_{k+1} = \frac{1}{4}F_k \end{cases}.$$

In the year $k = 0$, there are 8192 rabbits and 1024 foxes. When only four foxes survive, how many rabbits are there? (You must use linear algebra to solve this problem. Zero marks if you do not use linear algebra.)

Answer 7(b)	452

(Show your working below and on the next page.)

$$\begin{pmatrix} R \\ F \end{pmatrix}_{k+1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{4} \\ 0 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} R \\ F \end{pmatrix}_k \Rightarrow \begin{pmatrix} R \\ F \end{pmatrix}_n = \begin{pmatrix} \frac{1}{2} & -\frac{1}{4} \\ 0 & \frac{1}{4} \end{pmatrix}^n \begin{pmatrix} R \\ F \end{pmatrix}_0$$

$$\begin{vmatrix} \frac{1}{2}-\lambda & -\frac{1}{4} \\ 0 & \frac{1}{4}-\lambda \end{vmatrix} = 0 \Rightarrow (\tfrac{1}{2}-\lambda)(\tfrac{1}{4}-\lambda) = 0 \Rightarrow \lambda = \tfrac{1}{2} \text{ or } \lambda = \tfrac{1}{4}$$

$\lambda = \frac{1}{2} \Rightarrow 0x - \frac{1}{4}y = 0 \Rightarrow y = 0 \Rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is an eigenvector

$\lambda = \frac{1}{4} \Rightarrow \frac{1}{4}x - \frac{1}{4}y = 0 \Rightarrow x = y \Rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} \frac{1}{2} & -\frac{1}{4} \\ 0 & \frac{1}{4} \end{pmatrix}^n = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} (\frac{1}{2})^n & 0 \\ 0 & (\frac{1}{4})^n \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

(More working space for Question 7(b))

$$\therefore \begin{pmatrix} R \\ F \end{pmatrix}_n = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} (\frac{1}{2})^n & 0 \\ 0 & (\frac{1}{4})^n \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 8192 \\ 1024 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{7168}{2^n} + \frac{1024}{4^n} \\ \\ \frac{1024}{4^n} \end{pmatrix}$$

$$\therefore F_n = 4 \Rightarrow \frac{1024}{4^n} = 4 \Rightarrow n = 4$$

$$\therefore R_n = \frac{7168}{2^4} + \frac{1024}{4^n} = \underline{\underline{452}}$$

Question 8 (a) [5 marks]

Classify the linear systems of ordinary differential equations with matrices (i) $\begin{pmatrix} 1 & 3 \\ 1 & -2 \end{pmatrix}$, (ii) $\begin{pmatrix} 2 & -2 \\ 8 & 1 \end{pmatrix}$, (iii) $\begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix}$, (iv) $\begin{pmatrix} 0 & -2 \\ 8 & -0.1 \end{pmatrix}$, (v) $\begin{pmatrix} 0 & -5 \\ 5 & 0 \end{pmatrix}$.

Answer 8(a)	(i) SADDLE (ii) SPIRAL SOURCE (iii) NODAL SOURCE (iv) SPIRAL SINK (V) CENTRE

(Show your working below and on the next page.)

(i) Tr = −1, det = −5 ⇒ SADDLE

(ii) Tr = 3, det = 18, $Tr^2 - 4det$ = −ve ⇒ SPIRAL SOURCE

(iii) Tr = 4, det = 2, $Tr^2 - 4det$ = +ve ⇒ NODAL SOURCE

(iv) Tr = −0.1, det = 16, $Tr^2 - 4det$ = −ve ⇒ SPIRAL SINK

(V) Tr = 0, det = 25, $Tr^2 - 4det$ = −ve ⇒ CENTRE

Question 8 (b) [5 marks]

Humans and Klingons both live on Planet Zorg, but neither likes the other and both have a tendency to leave Zorg if they consider that the other side is too numerous. Despite this problem on Zorg, one million Humans and two million Klingons continue to move there from other planets as long as there are both Human and Klingon populations on Zorg. Suppose that we model this situation using the system of ordinary differential equations

$$\begin{cases} \frac{dH}{dt} = 5H - 4K + 1 \\ \frac{dK}{dt} = -H + 2K + 2\,, \end{cases}$$

where H and K denote the number in millions of Humans and Klingons respectively at any time t and time is measured in years. If at time $t = 0$, there are 50 million of Humans and $\frac{k}{6}$ million of Klingons living on Zorg, what is the minimum value that k must exceed in order that all Humans will be driven out of Zorg after some time?

Answer 8(b)	299

(Show your working below and on the next page.)

Let $\vec{r} = \begin{pmatrix} H \\ K \end{pmatrix}$, $B = \begin{pmatrix} 5 & -4 \\ -1 & 2 \end{pmatrix}$, $\vec{F} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$,

Then $\frac{d\vec{r}}{dt} = B\vec{r} + \vec{F} = B\vec{r} + BB^{-1}\vec{F} = B(\vec{r} + B^{-1}\vec{F})$

$\mathrm{Tr}B = 7$, $\det B = 6$, $\mathrm{Tr}^2 - 4\det = +ve \Rightarrow$ NODAL SOURCE

(More working space for Question 8(b))

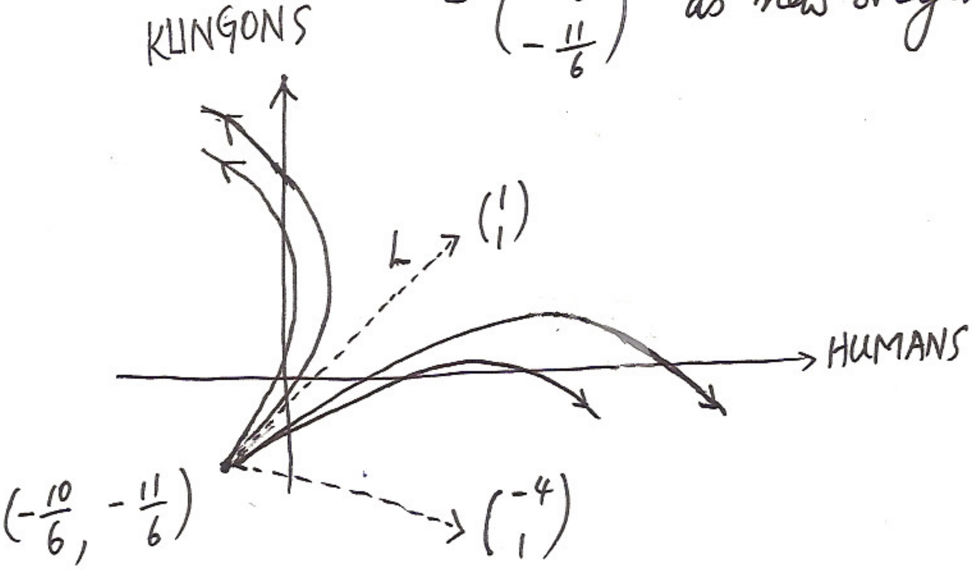
$$\begin{vmatrix} 5-\lambda & -4 \\ -1 & 2-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 7\lambda + 6 = 0 \Rightarrow \lambda = 1 \text{ or } \lambda = 6.$$

$\lambda = 1 \Rightarrow 4x - 4y = 0 \Rightarrow x = y \Rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector (slow direction)

$\lambda = 6 \Rightarrow -x - 4y = 0 \Rightarrow x = -4y \Rightarrow \begin{pmatrix} -4 \\ 1 \end{pmatrix}$ is an eigenvector (fast direction)

Translate axis to $-B^{-1}\vec{F} = -\frac{1}{6}\begin{pmatrix} 2 & 4 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -\frac{10}{6} \\ -\frac{11}{6} \end{pmatrix}$ as new origin



Equation of L is $K + \frac{11}{6} = H + \frac{10}{6}$

To drive H to zero $\Rightarrow K + \frac{11}{6} > H + \frac{10}{6}$

$\Rightarrow \frac{k}{6} > 50 + \frac{10}{6} - \frac{11}{6} = \frac{299}{6}$

$\Rightarrow k > \underline{\underline{299}}$