Ph.D. Qualifying Examination
Complex Analysis

1. Let $f(z)$ and $g(z)$ be entire analytic functions with $|f(z)| \leq |g(z)|$ for all z. Show that there is a constant c such that $f(z) = cg(z)$. Justify your arguments carefully.

2. Use Cauchy's residue theorem to evaluate the integral
$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+9)}.$$

3. State Rouche's Theorem. Use it to find the smallest integer n such that all the zeros of $z^5 + 3z^2 - 1$ lie in $|z| < n$. Justify your answer.

4. Let $f : U \to \mathbb{C}$ be analytic and $f(z) = u(x,y) + iv(x,y)$. Show that u, v and uv are all harmonic. Give an example of an f such that u^2 is not harmonic.

5. Find **all** conformal isomorphisms from the first quadrant of $\mathbb{C}$ to the open unit disk $D = \{z \in \mathbb{C} : |z| < 1\}$.

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